AB Calculus Quiz #13 NC
Integration Stuff
Dr. Wisniewski Spring 2020

Name Solution **Period** 3

Instructions: Solve each of the problems below. Please show your work (for partial credit) and box or circle your answers. A calculator is NOT permitted on this portion of the quiz.

1. (2 Pts) Let $f(x) = \int_{-5}^{x^2} \frac{1}{t^2+1}\,dt$. Find $f'(2)$.

$$f'(x) = \frac{1}{(x^2)^2+1} \cdot 2x = \frac{2x}{x^4+1}$$

$$f'(2) = \frac{4}{16+1} = \boxed{4/17}$$

2. (4 Pts) Find the average value of the function $f(x) = 16 - x^2$ on $[-4, 4]$. Find at least one value of x, call it $x = c$, for which $f(c) =$ the average value.

$$\bar{f} = \frac{1}{4-(-4)} \int_{-4}^{4} (16 - x^2)\,dx = \frac{2}{8}\int_{0}^{4} (16 - x^2)\,dx = \frac{1}{4}\left[16x - \frac{x^3}{3}\right]_0^4$$

(even)

$$= \frac{1}{4}\left[16(4) - \frac{4^3}{3}\right] = \frac{4^3}{4}\left[1 - \frac{1}{3}\right] = 4^2 \cdot 2/3 = \boxed{32/3}$$

3. (2 Pts) The average value of a continuous function $f(x)$ on $[3, 7]$ is 12. What is the value of $\int_3^7 f(x)dx$?

$$\bar{f} = 12 = \frac{1}{7-3}\int_3^7 f(x)\,dx = \frac{1}{4}\int_3^7 f(x)\,dx$$

$12 = \frac{1}{4}\int_3^7 f(x)dx$ mult both sides by 4

$$\boxed{48 = \int_3^7 f(x)dx}$$

4. (4 Pts) A large auditorium initially contains 132 people that received priority seating for a show (prior to the doors opening for general admission). At 8 pm, the doors open for general-admission seating and people enter the auditorium at the rate $R(t) = 10t - t^2$ measured in people/minute and where t represents the number of minutes past 8:00 pm. The doors close promptly at 8:10 pm and the show begins. How many people are in attendance for the start of the show? Show the work to support your answer.

Let $N(t)$ = # of people in and at time t where t represents the time since 8 pm.

$N'(t) = R(t)$ Net change thm

$$N(10) - \underbrace{N(0)}_{132} = \int_0^{10} R(t)\,dt$$

$$N(10) = 132 + \int_0^{10} (10t - t^2)\,dt = 132 + \left[5t^2 - \frac{t^3}{3}\right]_0^{10}$$

$$N(10) = 132 + \left[500 - \frac{1000}{3}\right] = 132 + \frac{1500 - 1000}{3} = 132 + \frac{500}{3}$$

298 or 299 depending on whether u round up or down.

5. (8 Pts) Evaluate each of the following integrals.

a. $\int \frac{x^2+5x-1}{x}\,dx = \int \left(x + 5 - \frac{1}{x}\right) dx$

$= \boxed{\frac{x^2}{2} + 5x - \ln|x| + C}$

b. $\frac{d}{dt}\int_2^8 e^x\,dx$ 0 deriv of a konstant = 0

c. $\frac{d}{dx}\int_5^x \ln t\,dt$ FTC part 2 $= \boxed{\ln x}$ No +C, No absval.

d. $\int_1^{e^2} \frac{1}{x}\,dx = \ln|x|\Big|_1^{e^2} = \ln|e^2| - \cancel{\ln|1|}^{0} = \ln e^2 = 2$